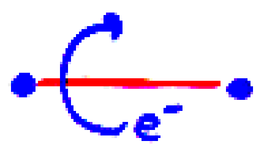
NUCLEAR MAGNETIC RESONANCE

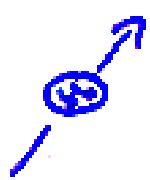
(GETTING ACQUAINTED)

MOST MAGNETIC PHENOMENA WE SEE IS A RESULT OF THE MOTION OF **ELECTRONS**

COIL OF WIRE



$^2\Pi_{1/2}$ NO



$\vec{\mu}$ OF THE ELECTRON ITSELF **SPIN!**

SPIN : MAGNETIC DIPOLE MOMENT RESULTING FROM ∢ MOM OF CHARGED PARTICLE

$$\vec{\mu} = \gamma \vec{J}$$

MAG MOMENT

GYROMAGNETIC RATIO

∢ MOM

γ IS A PROPERTY OF THE <u>PARTICLE</u>

LOTS OF PARTICLES HAVE SPIN

<u>NUCLEI</u>! <u>PROTONS</u>!

NUCLEAR MAGNETIC MOMENTS ARE $\approx$ 2000 TIMES <u>SMALLER</u> THAN ELECTRONIC ONES

→ USUALLY PERFORM NMR ON DIAMAGNETIC SAMPLES.

PLACE A PROTON IN AN EXTERNAL MAGNETIC FIELD, $\vec{H}_0$

$$\underset{\sim}{\mathcal{H}} = -\vec{\mu} \cdot \vec{H}_0 = -|\vec{\mu}||\vec{H}_0| \cos\theta$$

$$\vec{H}_0 \parallel \hat{z}$$

$$\vec{\mu} \cdot \vec{H}_0 = \underset{\sim}{\mu}_z H_0$$

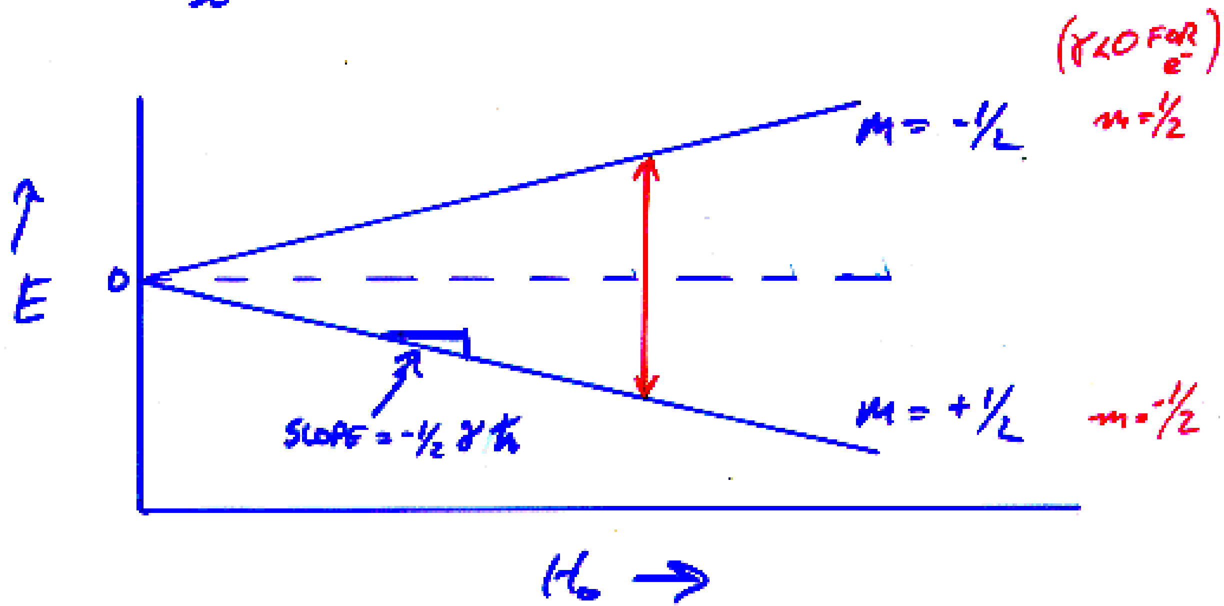
$$\vec{\mu} = \gamma \vec{J} \quad ; \quad \underset{\sim}{\mu}_z = \gamma \underset{\sim}{J}_z$$

Q.M. : $J_z |\psi\rangle = \hbar m |\psi\rangle$

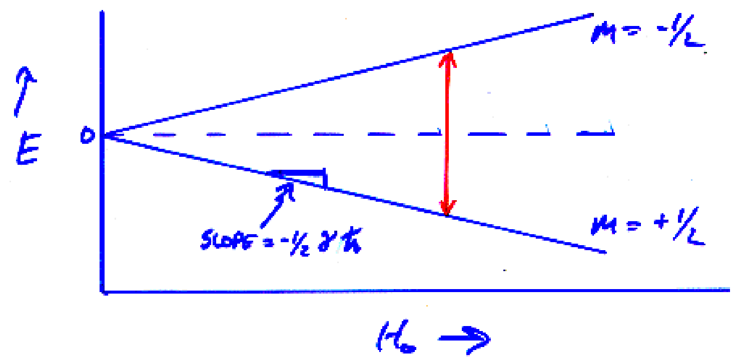
MAGNETIC QUANTUM NUMBER

$\pm 1/2$ FOR e^-, p

$$\langle \underset{\sim}{H} \rangle = E = -\gamma \hbar H_0 m \, ; \quad m = \pm 1/2$$



COULD MEASURE γ IF YOU COULD INDUCE A TRANSITION BETWEEN $m = +1/2$ AND $m = -1/2$ (A SPIN FLIP) WITH RADIATION



SPECTROSCOPY: LIGHT INTERACTING WITH MATTER

THIS IS A UNIQUE TRANSITION!
I.E. MAGNETIC DIPOLE NOT ELECTRIC DIPOLE

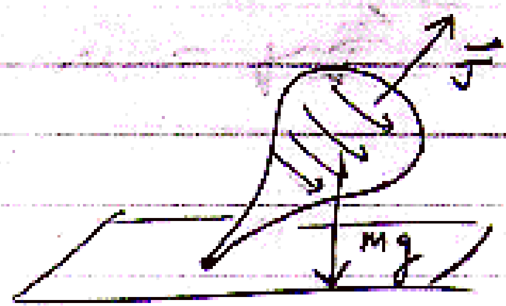
FREQUENCY?

$$\hbar\omega = E = \tfrac{1}{2}\gamma\hbar H_0 - \left(-\tfrac{1}{2}\gamma\hbar H_0\right) = \gamma\hbar H_0$$

$$\boxed{\omega = \gamma H_0}$$

CLASSICAL RESULT!

BUT SPINS WILL BEHAVE LIKE A TOP (REMEMBER WHAT THEY DO?)



$$\vec{\tau} \equiv \text{TORQUE} = \vec{r} \times \vec{F} = \frac{d\vec{J}}{dt}$$

THE TORQUE CAUSES THE TOP TO PRECESS ABOUT THE VERTICLE AXIS AT CONSTANT ANGLE, θ !

$\vec{J}$ CHANGES WITH TIME (EXTERNAL FORCE)

$\vec{J}$ VECTOR TRACES OUT A CONE!!

FOR A SPIN, THE TORQUE IS PROPORTIONAL TO $|\mu|$ AND $|H_0|$

$$\vec{\tau} = \frac{d\vec{J}}{dt} = \vec{\mu} \times \vec{H}_0$$

NOTE $\vec{\tau}$ IS $\perp$ TO $\vec{\mu}$ <u>AND</u> TO $\vec{H}_0$

FOR $\vec{H}_0$ INDEPENDENT OF TIME $|\vec{\mu}|$, θ CONSTANT IN TIME

$$\frac{d\vec{J}}{dt} = \vec{\mu} \times \vec{H}_0$$

$$\vec{\mu} = \gamma \vec{J}$$

$$\boxed{\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{H}}$$

NOTE $\mathcal{H} = -\vec{\mu} \cdot \vec{H}_0$; FOR TIME INDEPENDENT FIELDS ENERGY IS CONSERVED

SINCE WE KNOW THAT THE SPIN PRECESSES AT A CONSTANT ANGULAR FREQUENCY, WE WISH TO EXPRESS ITS MOTION IN A ROTATING COORDINATE SYSTEM TO SIMPLIFY MORE COMPLEX PROBLEMS

SOME MATH

CONSIDER A VECTOR, $\vec{F}(t)$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$\hat{i}, \hat{j}, \hat{k}$ ARE UNIT VECTORS IN SPACE

NORMALLY $\hat{i}, \hat{j}, \hat{k}$ ARE CONSTANTS (IN TIME) BUT WE WANT THEM TO ROTATE. THE LENGTHS OF THE UNIT VECTORS WILL BE CONSTANT. BUT THE WILL ROTATE AT CONSTANT ANGULAR VELOCITY

$$\frac{d\hat{i}}{dt} = \Omega \times \hat{i}$$

$\Omega = \measuredangle$ VEL OF COORD SYSTEM

$$\frac{dE}{dt} = \frac{dF_x}{dt}\hat{i} + F_x\frac{d\hat{i}}{dt} + \frac{dF_y}{dt}\hat{j} + F_y\frac{d\hat{j}}{dt} + \frac{dF_z}{dt}\hat{k} + F_z\frac{d\hat{k}}{dt}$$

$$\frac{dF}{dt} = \frac{dF_x}{dt}\hat{i} + F_x\frac{d\hat{i}}{dt}$$

$$+ \frac{dF_y}{dt}\hat{j} + F_y\frac{d\hat{j}}{dt}$$

$$+ \frac{dF_z}{dt}\hat{k} + F_z\frac{d\hat{k}}{dt}$$

$$= \frac{dF_x}{dt}\hat{i} + \frac{dF_y}{dt}\hat{j} + \frac{dF_z}{dt}\hat{k}$$

$$+ \Omega \times (\hat{i}F_x + \hat{j}F_y + \hat{k}F_z)$$

$$\left(\frac{d\vec{F}}{dt}\right) = \left(\frac{\delta \vec{F}}{\delta t}\right) + \underline{\Omega \times \vec{F}}$$

CHANGE OF ROTATING FRAME

CHANGE OF F IN ROTATING FRAME

APPLY TO $\boxed{\dfrac{d\vec{u}}{dt} = \vec{u} \times \gamma \vec{H}_0}$

$$\frac{\delta \vec{u}}{\delta t} + \vec{\Omega} \times \vec{u} = \vec{u} \times \gamma \vec{H}_0$$

$$\boxed{\frac{\delta u}{\delta t} = \vec{u} \times \left(\gamma \vec{H}_0 + \vec{\Omega} \right)}$$

MOTION LOOKS (IN ROTATING FRAME) LIKE A SPIN PRECESSING IN AN EFFECTIVE FIELD

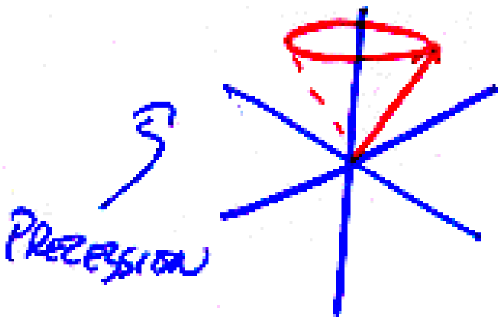
$$H_e = H_0 + \frac{\Omega}{\gamma}$$

MAKE $H_e = 0$? SURE !

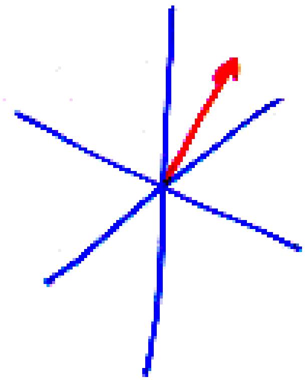
$$\vec{\Omega} = -\gamma \vec{H}_0$$

$\gamma H_0 \equiv$ LARMOR PRECESSION FREQ!

$\omega = \gamma H_0 \equiv$ RESONANCE CONDITION!

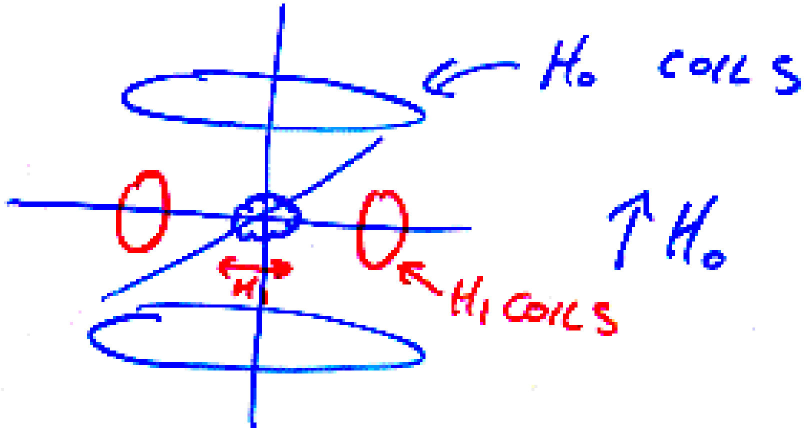


FIXED
FRAME



ROTATING
FRAME $\Omega = -\gamma H_0$

WHAT ABOUT ABSORB / EMIT ?



$$H_1 = H_{1x} \cos \omega t \; \hat{x}$$

CHOOSE FRAME ROTATING @ ω !

$$\frac{\delta \vec{\mu}}{\delta t} = \vec{\mu} \times \left[\underbrace{\hat{k}(\gamma H_0 - \omega)}_{\text{OLD}} + \underbrace{\hat{\imath}\,\gamma H_1}_{\text{NEW}} \right]$$

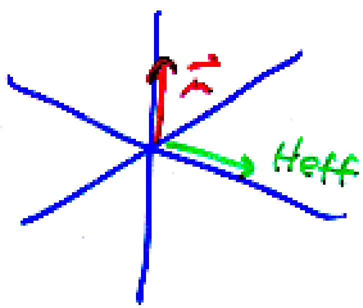
$$\frac{\delta \vec{\mu}}{\delta t} = \vec{\mu} \times \vec{H}_{eff}$$

$$\vec{H}_{eff} = \hat{k}\left(H_0 - \frac{\omega}{\gamma}\right) + \hat{\imath}\, H_1$$

TUNE H_1 TO RESONANCE

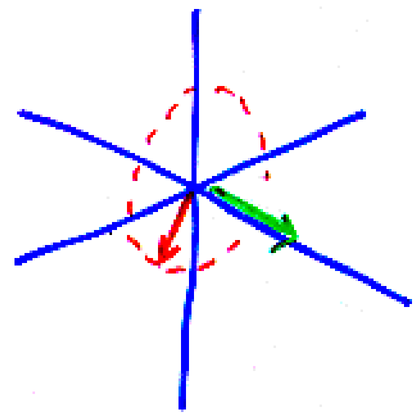
$$\omega = \gamma H_0$$

$$\vec{H}_{eff} = \hat{i} H_1$$



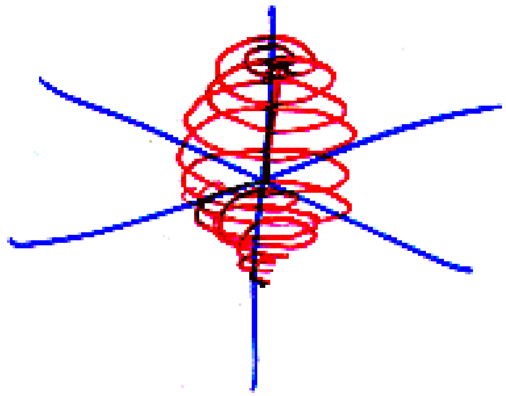
ROTATING FRAME

$t = 0$



t = LATER

$\vec{\mu}$ PRECESSES ABOUT H_{eff} IN ROTATING FRAME

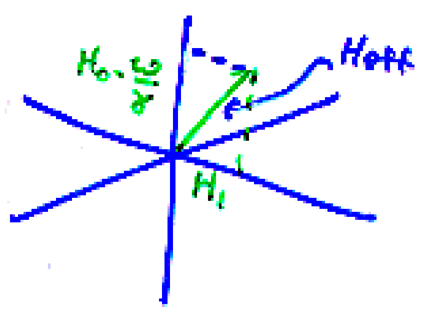


FIXED FRAME MOTION COMPLICATED!

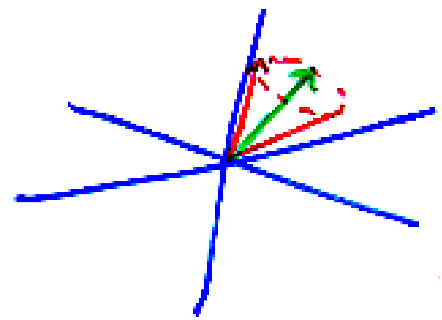
WHAT ABOUT NEAR RESONANCE

$$\omega - \gamma H_0 \neq 0 \ll 1$$

$$H_{eff} = \hat{k}\left(H_0 - \frac{\omega}{\gamma}\right) + iH_1$$



COMPONENTS OF H_{eff}



MOTION OF $\vec{\mu}$ IN ROTATING FRAME

$\vec{\mu}$ DOESN'T COMPLETELY TURN OVER

$\rightarrow$ LESS POWER ABSORBED

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